FAST FIGURING

FOR EXECUTIVES

(Present and Future)

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By

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Dedicated to

Mrs. Philip Klein

my sister-in-law, whose use of numbers is always refreshing.

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FAST FIGURING FOR EXECUTIVES

Foreword

FOR THE EXECUTIVE WHO LIKES TO READ A BOOK IN ONE PAGE

Cardinal Principles

1. Do not write a number more often than necessary.
2. Do not check your work if the error will show up before any damage is done.
3. Split numbers to advantage, for example, 26 may be considered as 25 and 1; 29 as 30 less 1.
4. Use small instead of large numbers when possible.
5. Adding, subtracting, multiplying and dividing are the bases of figuring. Learn when to substitute one for the other.

If you know all the following short-cuts, stop here!

1. Add by grouping numbers in ten (Chap. 3, No. 1).
2. Add and subtract pairs of numbers from left to right (Chap. 3, No. 4; Chap. 4, No. 4).
3. Multiplying by 9 is simply subtraction (Chap. 5, No. 2).
4. Two seconds is par for multiplying 35 × 35 (Chap. 9, No. 3).
5. 87 × 93 takes three seconds longer (Chap. 9, No. 1).
6. Divide by multiplying (Chap. 2).
7. An error of 3636 should be corrected easily. (Chap. 12. No. 8).
8. That 7854 × 9243 equal 72594522 is easily proved (Chap. 12, No. 9).

YOU ARE IMPORTANT

THIS little book is aimed at helping you—Executive, Present and Future—to put more work into the time spent at working.

It should help you save time; time for yourself, for your organization, for your associates. The extra time saved can be spent profitably with customers, in merchandising, billing, accounting, leisure, hobbies and even sleep!

Not only may you use these short-cuts yourself, they may be of value to a good many in important places in your organization. This book should stimulate you to develop and pass on ideas which you have used in your business.

Even if you find no immediate use for some of the short cuts, they should prepare you for advancement—for yourself or for your organization.

As you will see, there is competition even among the short-cuts offered. The faster of two methods should always be used after proficiency is acquired in both methods. Trial problems are offered, but much practice is usually necessary to develop speed.

The short-cuts apply to machine operations on adding machines and calculators as well as those done manually or mentally. When there is a choice between fast and faster an executive will determine whether to use a machine or do a problem manually. But often, machines are not available — in conferences or away from offices — and the executive should be able to solve a problem quickly.

The methods in this book were collected, discovered or developed and used by me in accounting, billing, buying, estimating, inventorying, payroll and selling.

They have actually been used; they have saved me time. I hope they help you.

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Chapter 1

JET ACTION SPEED

1. Do problems as fast as you can. Time saved can be used for making money (or having fun) in other ways!

2. Speed goes hand-in-hand with accuracy. Usually, anyone who is fast with numbers is also accurate.

3. Re-check your work most economically. Do not re-check when the time or money that may be saved by absolute freedom from errors is less valuable than the time spent in rechecking. (See Chapter 10.)

4. Develop speed by trying various methods of solving problems and using the fastest ones in the future.

5. Avoid taking time to write down a number when it is unnecessary. (See Chapter 5.)

6. Use the charts prepared for your industry. All sorts of tables are available, but if none has been prepared for your purpose make your own.

Chapter 2

COMPLEMENTS AND RECIPROCALS

A COMPLEMENT is a number which, added to its base, equals unity. For our purposes, unity is 1, 10, 100, 1000, .1, .01 etc.

Examples: (a) 1, 2, 3, 4, 5, 6, 7, 8 and 9 are complements of 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively. Added together, each pair of complementary numbers make 10.

(b) 1 and 99 added together make 100, so do 12 and 88; 25 and 75; 33 and 67; 40 and 60, etc. They are complements.

(c) 125 and 875 make 1000; 1111 and 8889 make 10,000, etc. They are complements.

(d) ⅛ and ⅞ added together make 1 and are complements.

(e) .25 and .75 added together make 1 and are complements. So are 33% and 67%, etc.

A RECIPROCAL of a number is unity divided by the number.

Examples: 1 divided by 5 = 1/5. 5 and 1/5 are reciprocals as are .03 and 33⅓, 4 and .25, 5 and .20, .008 and 125, etc.

Reciprocal numbers may be useful in all phases of arithmetic as you will see later. There is a shadow that accompanies every number and it may be easier to use the shadow (complement or reciprocal) rather than the original number itself.

Chapter 3

ADDITION

1. Add by grouping numbers together in tens and twenties.

g	f	e	d	c	b	a
3	4	7	3	2	8	4
5	2	8	8	4	3	2
2	4	2	2	1	7	9
8	8	9	4	7	6	4
1	2	1	8	6	8	8
4	5	3	8	7	2	2

a) 4, 2 and 4 (skipping 9) = 10, 8 and 2 make it 20 and 9= 29. Mark down 9 and carry 2.

b) 2 plus 8 = 10, 7 and 3 make it 20, plus 6 = 26, plus 10 (8 and 2) make 36, carry the 3.

c) 3, 2, 4 and 1 equal 10. 7, 6 and 7 equal 20, total 30. Carry the 3.

d) 3 plus 3 = 6, plus 10 (8 and 2) = 16, plus 20 (4, 8 and 8) = 36. Carry the 3.

e) 3 plus 7 = 10, 8 plus 2 make it 20, 9 plus 1 make it 30, plus 3 = 33. Carry the 3.

f) 4 plus 2 plus 4 = 10, with the carried 3 = 13, plus 10 (8 and 2) = 23 and 5 makes 28. Carry the 2.

g) 2 plus 3 plus 5 equals 10, 2 plus 8 makes it 20, plus 1 and 4 totals 25.

Answer: 25836069

2. Writing in the carry-over figure takes time. The practice can do no good unless the figures are re-added for errors but follow the practice if the column will surely be re-added by yourself or contains 5 or more numbers.

3. Learn groups of numbers (in any sequence).

a) 1, 3, 6 = 10; 2, 6, 2 = 10; 2, 3, 5 = 10; 3, 3, 4 = 10; 2, 4, 4 = 10 etc.

b) 6, 7, 7 = 20; 4, 7, 9 = 20 etc.

c) 1, 2, 3, 4 = 10; 1, 2, 3, 4, 5 = 15; 6, 7, 8, 9 = 30; 1, 2, 3, 4, 5, 6, 7, 8, 9 = 45.

4. Since it is faster to read and write from left to right, so it is faster to add from left to right. The mind can work faster than the hand even when it has to look ahead several digits.

Try these examples

8 9 4 3 2 8   3 9 7 8 9 4
3 8 9 7 5 6   2 4 9 7 1 8

It is more fun to add from left to right. But limit this method to two number additions.

5. Add in columns of 2 digits. This requires familiarity with numbers from 1 to 99, but with practice, it is as easy to add 82 and 97 as 8 and 9.

Example: 78, 42, 39
98, 76, 55
10, 88, 92

39 plus 55 equals 94 plus 92 equals 186. Mark down 86 and carry 1. 43 plus 76 makes 119 plus 88 equals 207. Mark down 07 and carry 2. 78 plus 2 equals 80 plus 108 equals 188. Answer 1880786.

6. Add the numbers as they are being written down. This is rather difficult when writing the numbers yourself, but when another person writes them it is comparatively easy to add the last row of digits; the last two rows of digits when they consist of 25, 50, 75c items. Try it the next time the grocery clerk lists your purchases.

7. Add long lists of numbers given orally by writing down only the cumulative sums. The answer is ready when the list is finished.

Example: 9 plus 9 plus 12 plus 42 plus 9   18
30
Answer: 81   72
81

8. Add by subtraction of complementary numbers.

Examples: 78 + 88 = 178 − 12 = 166
78 + 199 = 278 − 1 = 277

9. Multiplication is obviously the short-cut method of adding.

Example: 842 plus 842 plus 842 = 3 × 842 = 2526

10. When a number is duplicated often in a column, count the frequency of its appearance, multiply the frequency by the number and add the product to the rest of the column.

11. Oft-times when using an adding machine, two, three or more simple numbers may be added mentally and the sum entered on the adding machine.

PROBLEMS IN ADDITION

	a)	b)
#1	76438794	486324098
	31692013	247399212
	84371306	664782807
	53926549	313814955
	27784561	556433473
	35149862	214511645
#4	743984321	429853674
	494462929	854763589
#5	849732487	247324148
	941287341	643848924
	182452471	392284374

ANSWERS

#5 a) 1973472299 b) 1283457446
#4 a) 1238447250 b) 1284617263
#1 a) 309363085 b) 2483266190

Chapter 4

SUBTRACTION

1. Subtract by adding complementary numbers.

Examples: 112 − 87 = 12 + 13 = 25
842 − 198 = 642 + 2 = 644

2. Addition is usually easier than subtraction.

Example: $947.32
−18.95 (2% cash discount)
−26.72 (freight)

Method: 18.95 + 26.72 = 45.67
947.32 − 45.67 = 901.65

This is somewhat easier for bookkeepers, in particular, who want to know how much was deducted from a bill when payment was made.

3. One step further is to reduce the number of operations by subtracting and adding at one time.

Example:
249.68
\+ 819.04
− 4.99
− 16.40

Method: 8 + 4 = 12 − 9 = 3
6 + 0 − 9 = − 3 − 4 = − 7
write down 3 (complement of − 7) and carry − 1
9 − 1 + 9 = 17 − 10 (4 + 6) = 7
4 + 1 − 1 = 4
2 + 8 = 10

Answer: 1047.33

4. Subtract from left to right in 2 number problems.

Examples
872483 − 749878
76432 − 67894

PROBLEMS

#3	a)	10,864.32 + 8,257.91 − 217.29 − 165.16	b)	97,248.63 + 24,519.26 − 79,436.25 − 16,298.42
#4	a)	97,248.63 − 24,519.26	b)	79,436.25 − 16,268.42

ANSWERS

#3	a)	18,739.78	b)	26,033.22
#4	a)	72,729.37	b)	63,167.83

Chapter 5

MULTIPLICATION

1. The easy way to multiply by 10 is not

$$\begin{array}{r} 7854 \\ \times 10 \\ \hline \end{array}$$

but rather, simply to write down the number and add a zero to it.

e.g. 78540

Adding 2 zeros multiplies a number by 100; 3 zeros by 1000; 6 zeros by a million, etc.

2. Multiplying a number by 9 is easily done by adding a zero and then subtracting the original number.

Example: $7854 \times 9 = \begin{array}{r} 78540 \\ -\ 7854 \\ \hline 70686 \end{array}$

Subtracting is easier than multiplying.

3. Multiplying a number by a multiple of 9 (18, 27, 36, etc.) is done by multiplying the number by the multiple (of 9); then adding a zero and subtracting 10%.

Example: 7854×63

Solution: $7854 \times 7 = 54978$

$$\begin{array}{r} 549780 \\ -\ 54978 \\ \hline \end{array}$$

Answer: 494802

One multiplication is saved by this method.

Another Example: 31416×45

Solution: $31416 \times 5 = 157080$

$$\begin{array}{r} 1570800 \\ -\ 157080 \\ \hline \end{array}$$

Answer: 1413720

4. When a multiplier ends in 9, multiply by the next higher number and subtract the multiplicant.

Example: 7854 × 79
Solution: 7854 × 80 − 7854

```
 628320
−  7854
-------
 620466
```

Answer: 620466

5. Multiplying a number by 11

a) Add a zero and then add the original number.
Example: 7854 × 11 = 78540

```
  78540
+  7854
-------
```

b) Do the work mentally—same example.
Solution: Last number is 4; next number is 5 + 4 = 9; next number is 8 + 5 = 13. Mark down 3 and carry 1; next number is 7 + 8 + 1 = 16. Mark down 6 and carry 1. First number is 7 + 1 = 8.
Answer: 86394

6. Multiplying a number by 15 is done by adding a zero and then adding half the new number.

Example: 7854 × 15
Solution:

```
2) 78540
  +39270
  ------
  117810
```

Answer: 117810

7. Use reciprocal numbers, fractions and percentages.

Examples: 7854 × 25 = 785400 ÷ 4 = 196350
7854 × 125 = 7854000 ÷ 8 = 981750
7854 × 33⅓ = 785400 ÷ 3 = 261800
7854 × 16⅔ = 785400 ÷ 6 = 130900
64064 × 625 = ⅝ × 64064000 = 40040000

8. Use numbers with fewer digits as multiplier.

Examples: a) 872 × 3492 is better done

```
 3492    rather than     872
×872                   ×3492
-----                  -----
```

b) 87200 × 3494 may be

```
349400      or      3494
 ×872             ×87200
------            ------
```

9. With a simple multiplier such as 2, multiply from left to right. This practice is especially fast in figuring 2% cash discount in payment of bills.

Example: 348943724 × 2 = 697887448

10. When a number is used repeatedly in multiplication, make up a table of 1 to 9 times the number, then, simply copy and add instead of figuring each multiplication.

11. When multiplying by 7½ or 75, etc., multiply by 10 or 100 and subtract ¼

Example: 892 × 7½ = 8920 − 2230 (¼ × 8920) = 6690

12. Expand or contract a number to suit your convenience.

```
Example: 864 × 26 = 864 × 25 = 21600
              Plus  864 ×  1 =   864
                               -----
                               22464

Example: 864 × 49 = 864 × 50 = 43200
             Minus  864 ×  1 =   864
                               -----
                               42336
```

13. Cross multiplication permits you to multiply mentally. Only the result needs be listed instead of all the intermediate work.

Eample: 815 × 134

4 × 5 = 20. Put down 0 and carry 2

4 × 1 = 4 + 2 = 6 + 3 × 5 = 21. Put down 1 and carry 2

4 × 8 = 32 + 2 = 34 + 3 × 1 = 37 + 1 × 5 = 42. Put down 2 and carry 4.

3 × 8 = 24 + 4 = 28 + 1 × 1 = 29. Put down 9 and carry 2

1 × 8 = 8 + 2 = 10

Answer: 109210

PROBLEMS

2. a) 31,487,264 × 9
   b) 92,478,436 × 9
3. a) 31,487,264 × 81
   b) 92,478,436 × 36
4. a) 31,487,264 × 69
   b) 92,478,436 × 99
6. a) 31,487,264 × 15
   b) 92,478,436 × 15
7. a) 31,487,264 × 25
   b) 92,478,436 × 125
9. a) 31,487,264 × 2
   b) 92,478,436 × 2
11. a) 31,487,264 × 75
    b) 92,478,436 × 750

ANSWERS

2. a) 283,385,376
   b) 832,305,924
3. a) 2,550,468,384
   b) 3,329,223,696
4. a) 2,172,621,216
   b) 9,155,365,164
6. a) 472,308,960
   b) 1,387,176,540
7. a) 787,181,600
   b) 11,559,804,500
9. a) 62,974,528
   b) 184,956,872
11. a) 2,361,544,800
    b) 69,358,827,000

Chapter 6
DIVISION

1. The easy way to divide by 10,100 or 1000 is to strike off 1, 2, or 3 zeros.

Examples: 78540 ÷ 10 = 7854
785400 ÷ 100 = 7854
7854000 ÷ 1000 = 7854

Decimal points take care of missing zeros.

Example: 78540 ÷ 1000 = 78.54

2. Divide by multiplying with reciprocal numbers.

Examples: 785400 ÷ 25 = 7854 ×4
785400 ÷ 125 = 785.4 × 8
785400 ÷ 5 = 78540 × 2

3. When a number is used repeatedly as a divisor, determine its reciprocal by dividing it into 1 and then multiply by the many dividends.

Example:

Sales	157,432
Expenses—Wages	22,872
Expenses—Auto	5,623
Expenses—Rent	3,877
Expenses—Etc.	17,916
Expenses—Total	50,288

Problem: Find percentage of sales spent for various expenses.

Solution: Divide 157,432 into 1—*Answer:* .000006352. Multiply that by various expenses.

Answer: Wages 14.53% of sales
Auto Expenses 3.57%
Rent 2.46%
Total 31.94%

Note—This is obviously work for a calculating machine, but the method saves time, regardless.

4. When dividing by 7½ or 75 divide by 10 (or 100) and then add ⅓.

Example: 561 ÷ 7½ = 56.1 + 18.7 (⅓ × 56.1)
= 74.8

5. a) A number is divisible evenly by 3 when the sum of the digits in the number is divisible evenly by 3.

Example: 7854327258 is evenly divisible by 3 because 7 + 8 + 5 + 4 + 3 + 2 + 7 + 2 + 5 + 8 = 51 which is divisible evenly by 3.

b) A number is evenly divisible by 9 when the sum of the digits in the number is divisible evenly by 9.

Example: 3254789421 is evenly divisible by 9 because 3 + 2 + 5 + 4 + 7 + 8 + 9 + 4 + 2 + 1 = 45 which is divisible evenly by 9.

PROBLEMS

#2 a) 31416 ÷ 33⅓
b) 314160 ÷ 25
c) 3141600 ÷ 125

#3	Sales	3,333,333
	a) Gross Profit	1,298,921
	b) Wages	419,633
	c) Rent	150,000
	d) Total Expenses	733,333

#4 a) 3141600 ÷ 75

ANSWERS

#2 a) 942.48
b) 12566.4
c) 25132.8
#3 a) 39%
b) 12.6%
c) 4.5%
d) 22.%
#4 a) 41,888

Chapter 7
FRACTIONS

1. Know when to use them interchangeably with percentages and decimals. The following lists the ones found most helpful.

½ = .50 = 50%	⅛ = .125 = 12½%
⅓ = .33⅓ = 33⅓%	⅜ = .375 = 37½%
⅔ = .66⅔ = 66⅔%	⅝ = .625 = 62½%
¼ = .25 = 25%	⅞ = .875 = 87½%
¾ = .75 = 75%	1/9=.11-1/9=11-1/9%
⅕ = .20 = 20%	All Tenths
⅖ = .40 = 40%	1/11=.09-1/11=9-1/11%
⅗ = .60 = 60%	1/12=.08⅓=8⅓%
⅘ = .80 = 80%	1/15=.06⅔=6⅔%
⅙ = .16⅔ = 16⅔%	1/16=.0625 = 6¼%
1/7=.14-2/7=14-2/7%	

2. When multiplying by fractions it is usually faster to divide first by the denominator and then multiply by the numerator. Reason: Work with smaller numbers.

When the denominator does not divide evenly, however, it may be better to multiply first.

Examples: 210 × 5/7 = 30 × 5
200 × 5/7 = 1000 ÷ 7

3. When multiplying or dividing by compound fractions, there may be many short-cuts.

Examples: (a) 210 × 1⅕ = 210
+42

(b) 210 × 4⅘ = 1050 (5×210)
−42

(c) 210 ÷ 1⅖ = 210 ÷ 1.4
or 210 × 5/7

Chapter 8

PERCENTAGES AND DECIMALS

1. Know when to use them interchangeably with fractions (see table in chapter 7 #1).

2. Realize that multiplying by less than 1 gives a smaller number, but dividing by less than 1 gives a larger number.

Examples: 1000 × .5 = 500
1000 ÷ .5 = 2000

3. Eliminate the decimal hazard wherever possible by multiplying and dividing by 10, 100, 1000, etc.

Examples: (a) 78540 × .3 = 7854 × 3
(b) 7854 ÷ .33 = 785400 ÷ 33
(c) 7854000 × .125 = 7854 × 125
or 7854000 ÷ 8

4. In calculating 2% cash discount, follow this procedure.

(a) Multiply the hundreds of dollars in the principal by 2 to establish the dollars of discount.

(b) Multiply the dollars and tens in the principal by 2 to establish the cents of discount.

(c) 0 to 24 cents in the principal gives no discount; 25 to 74 cents gives 1 cent; 75 to 99 cents gives 2 cents.

Example: (1) 2% of $8754

Solution: 87 × 2 = 174.00
54 × 2 = 1.08

Answer: 175.08

Example: (2) 2% of 2496.85

Solution: 24 × 2 = 48.00
96 × 2 = 1.92
.85 = .02

Answer: 49.94

This is a variation of multiplying from left to right The method permits speedy calculations when the calculations are done entirely mentally.

5. The inexact but easy way to figure short term interest is to divide the year up into 12 months of 30 days. ⅙ annual rate of 6% gives 1% for every 2 months. 1/12 the annual rate of 4% makes 1/300 per month.

Examples: $1000 at 6% for 2 months is $10.00;
$1000 at 4% for 1 month is $3.33.

6. To determine costs, multiply list prices by the net cost percentage (complementary percentage).

Example: $225 less 40% = 225 × .60

7. Chain discounts are easily reduced to net cost percentage. Common trade discounts for your particular industry should be learned.

Example: $225—less 40%, less 20% = 225 × .48
Incidentally, that is easily figured at
½ of 225 less 2% of 225 112.50
− 4.50

Answer: 108.00

8. In determining a selling price that will gross 40% profit on the selling price, divide the cost by the complementary percentage.

> *Example:* What is the selling price of an item which costs $210, in order to make 40% of the selling price?
> *Solution:* Divide 210 by 60% or .60
> *Answer:* $350

PROBLEMS

#4 a) 2% of $31416.78
b) 2% of $7854.31

#6 a) $314.16 less 45%
b) $314.16 less 20%

#7 a) 8.40 less 40% less 15%
b) 8.40 less ⅓ less 25%
c) 8.40 less 88% less 16⅔%

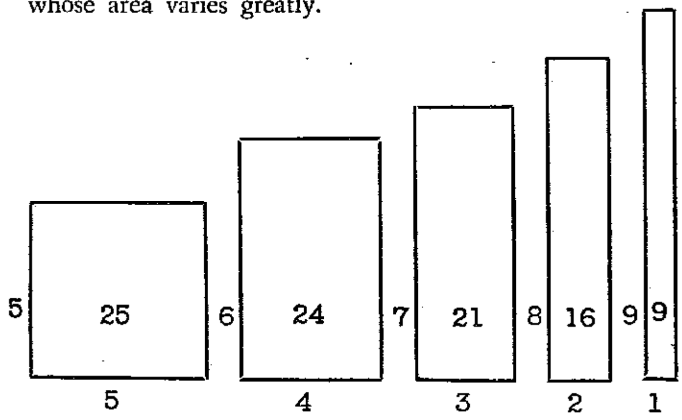
#8 a) 8.40 cost to gross 40% profit
b) 8.40 cost to gross 30% profit
c) 8.40 cost to gross 20% profit

ANSWERS

#4 a) $628.34
b) $157.09
#6 a) $172.79
b) $251.33
#7 a) $4.28
b) $4.20
c) $ .84
#8 a) $14.00
b) $12.00
c) $10.50

Chapter 9
SQUARING A NUMBER

Here are 5 shapes whose dimensions add up to 10 but whose area varies greatly.



Understanding these simple shapes helps to solve easily many multiplication problems such as 85×85; 88×92; 107×107.

1. Use the Formula: $(A + B) \times (A - B) = A^2 - B^2$.

If $A = 5$ and B varies with each block, apply the formula as follows:

$(5 + 1) \times (5 - 1) = 5^2 - 1^2$ or
$6 \times 4 = 25 - 1 = 24$
$(5 + 2) \times (5 - 2) = 5^2 - 2^2$ or
$7 \times 3 = 25 - 4 = 21$
$(5 + 3) \times (5 - 3) = 5^2 - 3^2$ or
$8 \times 2 = 25 - 9 = 16$
$(5 + 4) \times (5 - 4) = 5^2 - 4^2$ or
$9 \times 1 = 25 - 16 = 9$

Now for the larger numbers:

$101 \times 99 = 100^2 - 1^2 = 10{,}000 - 1 = 9999$
$92 \times 88 = 90^2 - 2^2 = 8100 - 4 = 8096$

In this first calculation the formula is used in multiplying 2 numbers which have an easily multiplied median (equi-distant from the two numbers).

2. Variation: To square a number.

$(A + B) \times (A - B) = A^2 - B^2$

easily becomes

$(A + B) \times (A - B) + B^2 = A^2$

Example: 78×78

Solution: Select an easily multiplied number near 78, namely 80, which is 2 more than 78.

$(78 + 2) \times (78 - 2) + 2^2 = 78^2$

$80 \times 76 + 4 = 6080 + 4 = 6084$

Answer: 6084

Another Example: 247×247

$(247 + 3) \times (247 - 3) + 3^2 = 247^2$

$250 \times 244 + 9 = 61000 + 9 = 61009$

Answer: 61009

3. Squaring a number ending in 5 is reduced to an even simpler formula.

Example: 75×75

Following the formula in number 2 of the previous page, it becomes $80 \times 70 + 25 = 5625$.

The Simplified Formula:

Multiply the number (without the last 5) by itself plus 1. Put down the answer and add two additional digits, namely 25.

Example: 75×75

$7 \times (7 + 1) = 7 \times 8 = 56\ (25)$
$= 5625$

Another Example: (125×125)

$12 \times (12 + 1) = 12 \times 13 = 156$
$(25) = 15625$

4. Multiplying two numbers whose left digits are the same and whose right digits add to 10 follows substantially the same simplified rule. Instead of adding 25, add the product of the right digits.

Example: 74×76.

Solution: $7 \times (7 + 1) = 7 \times 8 = 56$
$4 \times 6 = 24$

Answer: 5624

5. A different formula, $(A + B) \times (A + B) = A^2 + 2\,AB + B^2$, is used for squaring a number when you know the square of a nearby number.

Example: 26×26

Solution: 25×25 is known to be 625. Add 25 and 26 (or 51) to 625 and the answer is 676

PROBLEMS

#1 a) 82×78
b) 73×67
c) 121×119

#2 a) 31×31
b) 39×39
c) 99×99

#3 a) 95×95
b) 105×105
c) 85×85
d) 25×25

#4 a) 67×63
b) 101×109
c) 28×22
d) 34×36

ANSWERS

#1 a) 6396
b) 4891
c) 14399

#2 a) 961
b) 1521
c) 9801

#3 a) 9025
b) 11025
c) 7225
d) 625

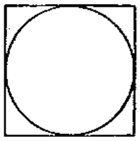
#4 a) 4221
b) 11009
c) 616
d) 1224

Chapter 10

PI

The Greek letter π (Pi) is the symbol for the ratio between the circumference of a circle and its diameter. It is usually expressed as 3.1416. 22/7 is the closest fraction.

For usual estimating, I suggest treating all circles as squares and then deducting ⅕; all balls as ½ the corresponding cubes.



a) The area of the circle is substantially ⅕ less than the area of the square. (2% excessive).
b) The circumference of the circle is substantially ⅕ less than the perimeter of the square.
c) The volume or area of a sphere is substantially ½ the volume or area of a corresponding cube. (5% insufficient).

Examples:

a) A circle 5″ in diameter has an area of 5 × 5 = 25 − ⅕ of 25 = 20 square inches. By the use of π (pi), the area is calculated 19.66 sq. inches.
b) The same circle has a circumference of 4 × 5 = 20 − ⅕ of 20 = 16 inches. By the use of π (pi), the circumference is calculated 15.708 inches.
c) A ball or sphere 5″ in diameter has an area of 6 × 25 sq. inches = 150 less 50% = 75 sq. inches. By the use of π (pi), the area is calculated 78.54 sq. inches.
d) A sphere 5″ in diameter has a volume of 5 × 5 × 5 = 125 − 50% = 62.5 cubic inches. By the use of π (pi), the volume is calculated 65.45 cubic inches.

Milady's hat size is the number of inches around her head, but a man's hat size is the same divided by Pi. Why??

Chapter 11

HOW MUCH PAINT?

1. The short cut in figuring square footage of walls is to add the length of all walls of equal height and to multiply that sum by the ceiling height.

Example: Room 8′ wide 12′ long and 10′ high
(2 × 8′) + (2 × 12′) = 40′
40′ × 10′ = 400 sq. ft.

2. The area of cylindrical tanks should be considered for most purposes as square tanks less ⅕.

Example: A tank 50′ in diameter and 100′ high with a flat circular top is to be painted. To calculate its paintable surface, treat it as a square tank. 50′ × 4 sides = 200′ × 100′ high = 20,000 sq. ft. + 50 × 50 (2500 sq. ft.) for the top = 22,500 sq. ft. Because it is a cylinder and not square, deduct ⅕ of 22,500 (4500) and the answer is 18,000 sq. ft.

2. a) The area of the surface of a ball or sphere is figured by treating it as a cube, less 50%. Thus, a 5′ ball, if it were square, would measure 25 square feet on each of its six sides. Therefore, 6 × 25 = 150, less 50% = 75 square feet, which is the surface area of the ball. An additional 5% may be added for greater accuracy. (See chapter 10.)

3. The use of the figure 231 cubic inches — contents of a gallon — is quite helpful in establishing this simple table for Paint coat thicknesses.

1″ thick covers 231 sq. inches or 1.6 sq. ft. to gal.
¼″ thick = 6.4 sq. ft. to the gallon

1/100″ thick = 160 sq. ft. to the gallon
1/250″ thick = 400 sq. ft. to the gallon
1/500″ thick = 800 sq. ft. to the gallon

PROBLEMS

#1. (a) 10′ wide × 15′ long × 12′ high
(b) 240′ wide × 360′ long × 18′ high

#2. How much paint (400 sq. ft. to gal.) for the sides and flat top of a tank 20′ in diameter, 50′ high?

#2a. How many gallons to paint a circular gas tank 25′ in diameter?

b. How much paint will that tank hold? A cubic foot is approximately 7½ gal.

#3. (a) Film thickness of paint covering 320 sq. ft. to the gallon.
(b) Paint 1/300″ thick covers how many sq. ft.?

ANSWERS

#1. (a) 600 sq. ft. #2a. 5 gallons (approx.)
(b) 21600 sq. ft. #2b. 58340 (approx.)
#2. 9 gallons (approx.) #3. (a) 1/200″
(b) 480 sq. ft.

Chapter 12

DOUBLE CHECK

It is the executive's job to determine whether the time spent in checking calculations is more or less important than the possible damage from errors which can be caught by re-checking.

Another factor is the knowledge which we gain from checking.

The rule to follow is substantially this: If damage can be done or loss incurred before work is checked by someone else or by bookkeeping procedure, regularly check your own extensions, additions, etc.

For example, when listing items purchased by a customer, the addition should be checked by the man writing or adding up the sale. Otherwise, no error would be discovered before the customer leaves the store.

Thanks to the great institution of Double Entry Bookkeeping, whenever most bookkeeping errors have been made, they show up. Debits then do not equal credits. Trial balances, profit and loss statements and balance sheets always come out even unless an error has been made. You can wait until a difference is shown before double-checking.

In other words, when work will be "proven" by something else, do not re-check until the proof shows something in error.

Short Cuts in Double-Checking

1. Know the amount of the error.
2. Check for that particular sum
(a) Scrutinize listings of that amount
(b) Devote no time in checking obviously correct figures; if the error is in dollars, do not check cents; if in thousands, do not check dollars.
(c) If divisible by 2, consider the possibility of having listed half the amount of error as a debit instead of credit (or vice versa).

(d) Recognize reversing (or transposition) errors e.g. 9247 instead of 9427. They always give an error that is divisible by 9. (More details later.)

3. Keep the columns you are comparing close together so that the eye does not travel far for comparisons.

4. Compare adding machine tapes rather than comparing previously written figures. Errors may have been made in running up machine tapes.

5. Check off marks or ticks need only be a short line or dot rather than a sprawling check mark which takes a longer time to write, is not as neat and mav be mistaken for a number.

6. When column A totals more than Column B (instead of being the same), work from column A. It is usually faster to find an error when items in the greater column A are checked against items in colum B.

7. Have a general idea of what the answer should be. Make the calculations with rounded figures. This usually eliminates gross errors.

Example: 7854 × 4.0942 is roughly 4 times 8000 and the correct answer should be near 32,000.

8. You may learn the following from the quotient of a reversing error divided by 9.

(a) In what digits the error has been made.
Example: Difference between 9427 and 9247 is 180. Divide 180 by 9 and the result of 20 indicates that there is no error in the right hand column (because of the 0 in 20) but that the error is in the tens and hundreds.
Another Example: Difference between 98743 and 89743 is 9000. When 9000 is divided by 9, the answer of 1000 shows that there is no error in the unit, 10 and 100 column (because of the 3 zeros in 1000) but exists in the thousands and 10 thousand column.

(b) The difference in the numbers reversed.
Example: 63 written instead of 36. The error shows up as 27. 27 ÷ 9 = 3. This shows that the error is in unit and tens column and also that the numbers reversed are 3 apart from each other. So one looks for reversal of 1 and 4, 2 and 5, 3 and 6, 4 and 7, 5 and 8, or 6 and 9.

(c) That the reversal was made between 2 numbers which are not next to each other. In this case, the answer after division by 9 is a number containing 2 positive figures.
Example: 28900 which should be 98200 shows error of 69300. 69300 divided by 9 equals 7700. We know from (a) and (b) above that there is no error in the unit and 10 columns and the difference between the numbers reversed is 7. But the double 7 shows that the error is not in the 3rd and 4th columns but rather in the 3rd and 5th columns.

9. Although it does not catch all possible errors, a rapid check of multiplication calculations is made by adding the value of all the digits involved.

Example: 7854 × 9243 = 72594522
Proof: 7 + 8 + 5 + 4 = 24; 2 + 4 = 6
9 + 2 + 4 + 3 = 18; 1 + 8 = 9
Then Multiply 6 × 9 = 54; 5+ 4 = 9
7 + 2 + 5 + 9 + 4 + 5 + 2 + 2 = 36; 3 + 6 = 9
It proves correct.

10. Likewise for division:
87266694 ÷ 827 = 105522
8 + 7 + 2 + 6 + 6 + 6 + 9 + 4 = 48;
4 + 8 = 12; 1 + 2 = 3;
8 + 2 + 7 = 17; 1 + 7 = 8
1 + 0 + 5 + 5 + 2 + 2 = 15; 1 + 5 = 6
6 × 8 = 48; 4 + 8 = 12; 1 + 2 = 3
It proves correct.

10a) Division: When there is a remainder, add the digit values of the remainder.

7843 ÷ 89 = 88-11/89
7 + 8 + 4 + 3 = 22; 2 + 2 = 4
8 + 9 = 17; 1 + 7 = 8
8 + 8 = 16; 1 + 6 = 7
8 × 7 = 56; 5 + 6 = 11; 1 + 1 = 2
Remainder 11; 1 + 1 = +2
—
4

It proves correct.

PROBLEMS

#8. Give two reversal errors which may have caused the following differences:
(a) 45 (c) 7200
(b) 630 (d) 1818

#9. Prove:
(a) 984732 × 14832 = 14605545024
(b) 3417 × 2426 = 8289642

#10. Prove:
(a) 3478178 ÷ 314 = 11077
(b) 9749536 ÷ 4798 = 2032

#10a. Prove:
(a) 9145627 ÷ 314 = 29126-63/314
(b) 41286325 ÷ 1042 = 39622-201/1042

ANSWERS

(#8)
#1 #2
(a) 50 — 05; 94 — 49
(b) 700 — 070; 810 — 180
(c) 8000 — 0800; 9198 — 1998
(d) 2020 — 0202; 7531 — 5713

(#9)
(a) 9 = 9
(b) 3 = 3

(#10)
(a) 2 = 2
(b) 7 = 7

(#10a)
(a) 7 = 7
(b) 4 = 4

Chapter 13

GENERALITIES

1. Multiplication tables are commonly known up to 12 × 12. Extend your knowledge as far as possible, at least up to 15 × 15.

2. Look at a number once and remember it. Repeating a number aloud may help. Concentrate on remembering the dollar portion of a long number and the cents will be recalled without effort. Aldous Huxley's "Art of Seeing" is a very interesting presentation of this subject.

3. Use all the mechanical means available if they will save time or eliminate the possibility of errors. Adding machines, calculators, slide rules, etc., are usually helpful.

4. Expand, by practice, your ability to make calculations without the aid of pencil and paper. The mind thinks faster than the hand can write.

5. Write numbers down only once.

Examples: 7854 × 210 should not be:

```
  7854
×  210        But rather    78540
------                     15708
 78540                    -------
15708                     1649340
-------
1649340
```

7854 × 16 should not be:

```
  7854
×   16        But rather    78540
------                      47124
 47124                     ------
 7854                      125664
------
125664
```

The multiplier may be written down off to the side somewhere.

6. Simplify keeping check book records. To calculate a balance, add all deposits to the old balance and subtract the sum of all the checks drawn. It is not usually necessary to make a deduction every time a check is drawn. In the case of three to the page check books, it is not necessary to add three checks and subtract every group of three.

7. Accountants know that it is not usually necessary to enter in purchase books every individual purchase. Bills from vendors may be grouped and one instead of many postings may take care of a month's purchases from one supplier.

8. It is usually easier to add than to subtract; to subtract rather than multiply and finally to multiply than to divide. However, many short cuts use one instead of the other calculations.

9. It is usually quicker to work with small numbers at first and to leave the large numbers until later operations.

Example: $12 \times 4864 \times 9$;
$9 \times 12 = 108 \times 4864$

10. In writing out numbers in checks, contracts, etc., it is faster to write 1100 to 1999; 2100 to 2999 etc., to 9100 to 9999 by eliminating the word thousand and writing hundreds, e.g. 1125 is Eleven Hundred Twenty-Five rather than One Thousand, One Hundred Twenty-Five.

Chapter 14
CONCLUSION

You have probably gained from what I have written that I like numbers. I do!

I have found them dependable, stable, correct and final.

They have helped train me to "come to the point." Out of a morass of facts jumps one useful problem or one pertinent equation.

Not only are they useful tools, but their stability helps keep us a little saner in this flighty world.

If the ideas expressed in this book are not clear, please feel free to communicate with me. I am very anxious to learn other short-cuts which have been helpful to others.

✓ ✓ ✓